p-Adic physics below and above Planck scales

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We present a rewiew and also new possible applications of p-adic numbers to pre-spacetime physics. It is shown that instead of the extension $I\!R^n \to {Q_p}^n$, which is usually implied in p-adic quantum field theory, it is possible to build a model based on the $I\!R^n \to Q_p$, where p=n+2 extension and get rid of loop divergences. It is also shown that the concept of mass naturally arises in p-adic models as inverse transition probability with a dimensional constant of proportionality.

I. INTRODUCTION

The results of any physical measurements are always expressed in terms of *rational* numbers: since any measuring equipment works with finite precision, bounded at least by the quantum uncertainty principle. The next step — to the field of *real* numbers — is a theoretical abstraction valid only at large classical scales. The geometry of the Planck scales [1] is, of course, unknown but there is a strong theoretical bias that the geometry of these scales should be non-Archimedian. Indeed, who and how could compare two distances then?

It has now come to be realised that the smooth spacetime continuum that is used even in quantum field theory is an approximation [2,3]. Space-time at the small scales appears to be granular. Indeed the very concept of the usual space and time in quantum theory is questionable [4,5]. The fact that time itself would have a minimum irreduceable unit, the chronon, was proposed quite sometime back [6,7]. These ideas find a culmination in the recent model of the quantum-mechanical black holes (QMBH) [8,9]. In this model the Compton wavelength and the corresponding time interval which is the same as the minimum time interval, the chronon refered to arise as the natural space time cutoffs – the Planck scales being a special case thereof. It is also worth noting that as pointed out by Wheeler [1] and others, it is a misconception to think that space and time are on the same footing. Rather our perception and hence description of the universe is one of "all space" at "one instant", that is effectively time is a parameter. Space and time are on the same footing in a stationary scenario, that is for a closed system as a whole [10,11]. This is brought out for example in the fluctuating foam like structure of space time at Planck scales [1].

Besides,there is a more rigouros (or may be more metaphysical) requirement that physical laws should be expressed in a coordinate-free language, i. e. as relations between some objects and sets (See e. g. [13] for a comprehensive review). This geometry — the quantum topology — is unknown as yet, and it is more philosophical than a physical question whether or not it can be discovered in detail. However it seems quite reasonable that it should be non-Archimedian, for there is no way to compare the distances at Planck scales. There are different models leading to non-Archimedian geometry. To some extent, we may say that the basic idea is that any set of objects can be labeled by an infinite series of integers. In p-adic models it is believed that p-adic numbers play an important role at the fundamental scales.

From the first glimpse it may seem that a p-adic number

$$x = \sum_{k=k_{min}}^{\infty} a_k p^k, \quad 0 \le a_k < p,$$

(where p is prime) is just another representation of an infinite sequence $\{a_k\}_{k_{min}}^{\infty}$. In fact, is there any difference whether p is a prime or just an integer? Physical intuition suggests no difference here. However, the mathematical truth is that only prime integers label different topologies available at continuum [14]. This fact seems to be of great physical importance. One may play with mathematics and methaphysics of the Planck scales as long as possible, but the results of all physical observations are obtained at the energies much below the Planck ones, therefore the correspondence between the existing and well tested theories (such as quantum electrodynamics) and that from the waiting list of quantum gravity can be checked for correspondence only in a continuous limit. (This however does not prohibit pure theoretical studies at Planck scale energies, which may lead to some cosmological consequences.)

The paper is organized as follows. In *section 2* we recapitulate the basic facts about p-adic numbers. In *section 3* we give a critical review of quantum models constructed on the D-dimensional p-adic space ${Q_p}^D$. In *section 4*

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we present our geometric approach to p-adic quantum field theory and its possible links to the space or space-time metric. Some possible consequence of all these things are presented in *last section*.

II. BASIC FACTS ABOUT P-ADIC NUMBERS

The results of any measurements, as was already mentioned, can be expressed in terms of rational numbers. The construction of a theoretical model, first of all a description in terms of differential equations, ultimately requires an extension of this field. The first extension (made by Dedekind, viz. equivalence classes) means the incorporation of rational numbers. It is a completion of the field $\mathbb{Q}$ with respect to the standard norm $\mathbb{Q} \stackrel{|\cdot|}{\rightarrow} \mathbb{R}$. This completion however does not exhaust all possibilities. It is also possible to extend the field $\mathbb{Q}$ using the *p-adic norm* $|\cdot|_p$ (to be explained below):

$$\mathbb{R} \stackrel{|\cdot|}{\leftarrow} \mathbb{Q} \stackrel{|\cdot|_p}{\rightarrow} Q_p. \tag{1}$$

This completion is called the field of p-adic numbers Q_p. No other extensions of $\mathbb{Q}$ except these two exist due to the Ostrovski theorem [15].

The p-adic norm $|\cdot|_p$ is defined as follows. Any nonzero rational number $x \in \mathbb{Q}$ can be uniquely written in the form

$$x = \frac{m}{n} p^{\gamma}, \tag{2}$$

where integers m and n are not divisible by the prime integer $p \neq 1$, and γ is an integer. The decomposition (2) provides a possibility to supply the field $\mathbb{Q}$ with the norm

$$|x|_p = \left| \frac{m}{n} p^{\gamma} \right|_p \stackrel{def}{=} p^{-\gamma}, \qquad |0|_p \stackrel{def}{=} 0, \tag{3}$$

different from the standard one. The algebraic closure of the field $\mathbb{Q}$ in the norm $|\cdot|_p$ forms the field of p-adic numbers Q_p.

Any p-adic number can be uniquely written in the form

$$x = \sum_{n=k_{min}}^{\infty} a_n p^n, \qquad a_n \in \{0, 1, \ldots, p-1\}, \quad k_{min} > -\infty. \tag{4}$$

It is easy to check that $|xy|_p = |x|_p |y|_p$, but $|\cdot|_p$ is stronger than $|\cdot|$:

$$|x+y|_p \le max(|x|_p, |y|_p) \le |x|_p + |y|_p \tag{5}$$

and induces a non-Archimedian metric

$$\begin{aligned} d(x,y) &:= |x-y|_P \\ d(x,z) &\le max(d(x,y), d(y,z)) \le d(x,y) + d(y,z), \end{aligned} \tag{6}$$

often called an *ultrametric* [16]. With respect to the metric (6) the Q_p becomes a complete metric space. The maximal compact subring of Q_p

$$\mathbb{Z}_p = \{x \in Q_p : |x|_p \le 1\} \tag{7}$$

is referred to as *a set of p-adic integers*. The field Q_p admits a positive Haar measure, unique up to normalization

$$d(x+a) = dx, \quad d(cx) = |c|_p dx, \quad x, a, c \in Q_p. \tag{8}$$

The normalisation is often chosen as $\int_{\mathbb{Z}_p} dx \equiv 1$.

The geometry induced by the distance $|x-y|_p$ is quite different from the Euclidean one: all p-adic triangles are equilateral; two p-adic balls may either be one within another or disjoint.

There is no unique definition of differentiation in the field Q_p, but the Fourier transform exists and is used in p-adic field theory to construct the (pseudo-)differential operator

$$\nabla \phi(x) \rightarrow |k|_p \tilde{\phi}(k).$$

The construction of the p-adic Fourier transform is essentially based on the group structure of the field Q_p, viz. the group of additive characters

$$\chi_p(x) := \exp\bigl(2\pi\imath\{x\}_p\bigr), \quad \chi_p(a+b) = \chi_p(a)\chi_p(b),$$

(where $\{x\}_p$ denotes the *fractional part* of x: $\{x\}_p = a_{min}p^{k_{min}} + \ldots + a_{-1}p^{-1}$), is used to construct the Fourier transform

$$\tilde{\psi}(\xi) = \int_{Q_p} \psi(x)\chi_p(\xi x)dx, \quad \psi(x) = \int_{Q_p} \tilde{\psi}(\xi)\chi_p(-\xi x)d\xi. \tag{9}$$

The n-dimensional generalization is straightforward

$$Q_p \to Q_p{}^n, \quad x \to (x_1,\ldots,x_n), \quad \xi \to (\xi_1,\ldots,\xi_n), \quad \xi x \to (\xi,x) = \sum_i \xi_i x_i.$$

III. P-ADIC QUANTUM FIELD THEORY

As any field theory starts from the action functional $S[\phi,\partial\phi,\ldots]$, so does the p-adic theory. At the first glance, it is just a bare substitution of the numeric field $I\!R^D \to Q_p{}^D$, and hence for the generation functional $Z[J] \to Z[J]|_{Q_p{}^D}$:

$$\begin{array}{lcl} Z[J]|_{Q_p{}^D} &=& \int \mathcal{D}\phi \exp\Bigl(\int_{Q_p{}^D} J\phi d^Dx + S[\phi]\Bigr) \\ S[\phi] &=& \int_{Q_p{}^D} \mathcal{L}(\phi,\partial\phi,\ldots)d^Dx, \end{array} \tag{10}$$

where all integrations in (10) are taken in the Fourier space, as in usual quantum field theory. Referring the reader to [17,18] for the detailed account of the Green functions and the Feynman expansion calculation, we have only to note that the theory (10) inherits (of course in milder form) the divergences of the loop integrals. The integral

$$\int_{Q_p^D} \frac{d^Dk}{|k^2|_p + m^2}$$

is divergent for $D \geq 2$.

Up to now we did not touch upon the question *why* it may be possible to substitute a numeric field in a field theory (scalar for simplicity) defined over a (pseudo-)Euclidean space by means of a certain action functional. First, it can be easily noticed that the availability of the extension $I\!\!\!Q \to Q_p$, even taken together with the principles of quantum field theory, does not suggest any particular value of p. Fortunately, as it follows from number theory, if all prime bases are taken togeter their collection resembles the field of real numbers, viz.

$$\prod_{p\in prime} \chi_p(\omega t - kx) = \exp(2\pi\imath(\omega t - kx))$$

and in this sense the *real free particle is a product of an infinite number of p-adic plane waves.* The other but similar argument, which has caused a deep interest in p-adics, is that the formulae for the string amplitude have the same form as p-adic adelic products.

IV. GEOMETRIC APPROACH TO P-ADIC QUANTUM FIELD THEORY

The question, from what and how our Universe and hence spacetime have emerged is a deep one. The standard tool of modern physics applied to this problem is a retrospective analysis. Starting from todays structure of the Universe we extrapolate its geometrical properties back in time using the results both general relativity and particle physics. The extrapolation stops at the Planck scales, where the problem whether or not we can go further down with our metric theories stops our consideration. However, we can turn the problem upside down. If the Universe *has emerged*, it should emerged from One *something.* At the same time a large variety of different objects exist now. So, there is a problem of how many could emerge from one. It is not only a philosophical, but also a physical problem.

If we reject the idea of *a priori* background space, then there is no geometrical space when there is only one object, and there is no time without a changing variety of material objects. So the *appearence of space and time requires*

certain relations between objects. We may think at least of two kinds of such relations: (i) the relations between objects belong to a certain set, and (ii) between "parents" and "descendants". The latter type of relations provides the multiplication of the initial set. At quantum level the "parent" does not exist any longer after giving ancestors, e. g. $\gamma \to e^+e^-$.

To start with our geometrical approach, let us consider a two dimensional sphere S^2 and find out what other coordinates except for well known ones (θ, ϕ) can be used to label the points on the sphere. To do this, we use the fact that an n-dimensional sphere can be considered as a boundary of an $(n+1)$-dimensional simplex. For two dimensions this implies that we can just take a 2D simplex — a triangle, — identify its vertices ($A = B = C$, see Fig.1), glue edges, and then getting a 3D simplex put it onto the sphere S^2.

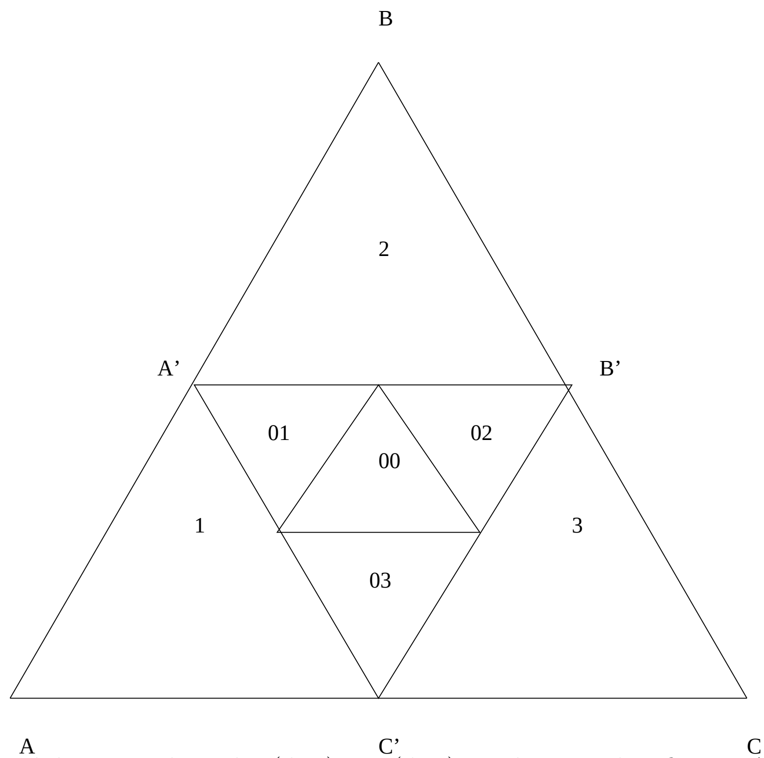

FIG. 1. The partition of a d-dimensional simplex (d=2) into (d+2) equal parts. Identification A=B=C and gluing edges are performed to put it over the sphere S^2.

We can divide our simplex into p=4 equal equilateral triangles and label them by 0,1,2,3. The initial triangle is therefore subdivided into $p = d + 2$ equal parts; since the same procedure can be also applied to its parts we can move down ad infinum and label these parts as ($a_0, a_1, \ldots$, where $0 \le a_i < p$). If we ascribe a measure $\mu(T_0) = 1$ to the initial triangle, when at first stage we have p triangles of the measure $1/p$ eacn, at the second stage p^2 triangles of the measure $1/p^2$ and so on. To meet this natural definition of the measure, we can label the sequences $\{a_i\}$ by p-adic fractional numbers $x \in \mathbb{Z}_p^{-1}$.

Then the measure of the (0,1) triangle for instance, (see Fig.1) labeled by $x = 1p^{-2} + 0p^{-1}$, will be $\mu(0,1) = |x|_p^{-1} = p^{-2}$. One may also label the multiplicative partition process by p-adic integers $x \in \mathbb{Z}_p$. In this case the measure can be taken as $\mu(x) = |x|_p$. In any case it is possible to locate any point of the sphere by an infinite sequence of its triangle vicinities of diminishing measure, which are in one-to-one correspondence to the space of p-adic integers $\mathbb{Z}_p$.

Here we would like to touch upon the question of the physical origin of that we call a *continuous manifold*. If there are only a few objects (with some relations between them) we cannot speak of a manifold. For example, the partition of a triangle seen above could be to some extent identified with 3 quarks (and something inbetween them) which constitute a nucleon, but nothing can be asserted about a continuous space here. If a collection of many more objects is considered and there is a dense web of relations between them, then we can start a continuous approximation – a manifold.

If we believe that the basic principles of quantum field theory developed for Euclidean space are also valid for such nets we can apply the Green functions, loop integrals etc., but there is no general need to project a D-dimensional Euclidean space or a D dimensional sphere onto this network (to that labeled by p-adic numbers, in particular). All

matrix elements, the Green functions etc. can be evaluated on the Q_p which label the triangle partitioning of the two-dimensional sphere S^2, the same to some extent can be done by the extensions $S^D \rightarrow Q_p, p = D+2$ for higher dimensions. As an example let us consider the loop integral

$$\int_{\mathbb{Z}_p} \frac{dk}{|k|_p^2 + m^2} = \sum_{\gamma=-\infty}^{0} \int_{S_\gamma} \frac{dk}{|k|_p^2 + m^2} = \left(1 - \frac{1}{p}\right) \sum_{\gamma=-\infty}^{0} p^\gamma \frac{1}{p^{2\gamma} + m^2} \tag{11}$$

The inegral is evidently finite for any p. So we have just demonstrated the possibility of getting rid of loop divergences in p-adic field theory.

It is worth noting at this point that the self interaction which leads to infinities both in the classical theory of the electron and in QED, is infact perfectly meaningful and gives rise to the mass in the theory of the QMBH refered to earlier [11], as we will see later.

Now let us give a geometrical interpretation to the integration formula (11). The sum of its right side it is due to the isotropy of the integrand $f(k) = \frac{1}{|k|_p^2 + m^2}$ which depends on the $|k|_p$ only, but not on the k itself: $f(k)$ is a constant on p-adic circles $S_\gamma = \{x \in Q_p \| x|_p = p^\gamma\}$. Since each of these circles contains exactly $(p-1)$ points, the sum can be easily evaluated. This is an analogue of the integration of the spherically symmetrical functions in Euclidean space $d^3r \rightarrow 4\pi r^2 dr$.

The case of the p-adic partitioning of the sphere may first seem very specific. More generaly, any point in a physical space can be located by the nested set of its vicinities:

$$\ldots \subset V_1 \subset V_0. \tag{12}$$

The difference between two nested vicinities (balls) can be referred to as a circle $S_\gamma = V_{\gamma-1} - V_\gamma$, exactly as in the p-adic case. The spaces V_i of the nested family (12) do not form a σ-algebra, their differences S_i do, and that is why the integration is defined on $\sum_i S_i$.

Up to now we dealt with mathematical constructions only. The question is are there any physical counterparts? Evidently the description of the Universe in terms of locally Euclidean coordinates is not unique. The location of any physical object in the Universe can be also defined by the nested set of vicinities (12): viz. any quark is inside a nucleon/meson, any nucleon inside an atom, atom inside a galaxy, and so on up to the whole Universe (V_0), which contains everything by definition:

$$x \in V_m \subset V_{m-1} \subset \ldots \subset V_0.$$

The intersection of each two of these sets should coincide with one of them: they can not have a partial intersection. The other property, which is physically required, is that the sets V_i, and hence their differences S_i, should have some discrete symmetry. If all they were structureless, it would be only one quark, one atom, one galaxy etc. in the Universe.

The situation is much like the structure of Q_p, but is not completely identical to it. There is also the question of how distant galaxies of our present Universe could be related to a non-Archimedian p-adic toy model. The answer may be as follows. The present state of the Universe is a result of expansion which has been taken place after the Big Bang. Before the Big Bang it might have been only a network of relations between some primary objects emerged from the primary One. Then, due to multiplicative processes the number of the objects (particles) increased greatly, but some of the relations between them were inherited and manifest themselves even in large-scale structures. The distance between different objects, even between galaxies, may therefore be measured not only by traveling light waves, but also by the level of their common ancestor in the evolution process.

It is important to note here, that two simple asumptions: (i) that the set of vicinities $\{V_i\}$ with which we locate any object is countable and (ii) that the union $\mathcal{H} = \overline{\cup_i V_i}$ is a complete Haussdorff space, immidiately lead to the conclusion that $\mathcal{H}$ should be *metrizable* due to the Urysohn lemma (see e.g [12]). Illustrative examples of a similar type metrisability has been given for different evolutional classifications of the species, for data analysis. But since the results are general, this can be also applied to fundamental structures of quantum space-time.

a. Time. We have already commented on the role of time The role of time in the above considered model may be two-fold. *First, it may be taken as just an evolution parameter*, related to some clocks which are either external to the system or, being installed at some deeper level of the same system, have no effect on the dynamics of the observed level. To some extent, it is similar to an atomic clock used to study the dynamics of terrestrial objects. This we will call the classical, or nonrelativistic time point of view.

The other possibility, we call it *quantum time point of view*, is to understand time as a discrete coordinate which counts the number of branching points, and can be properly defined only between objects (=events) of the same evolution branch. We call it time-like distance.

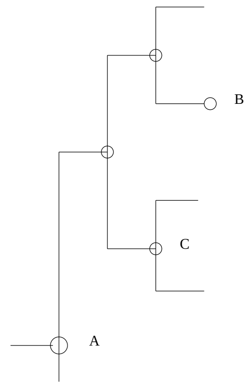

FIG. 2. An illustrative binary tree structure of the metric: The distance between two events (objects) is proportional to the age of their common ancestor, i.e. to the time elapsed since they branched off. $T_{AB} = 3, T_{BC} = 2$.

The illustration is presented in Fig.2. The time distance $T_{AB} = 3$ there. The distance of this kind can be also defined between two events on different branches, i. e. between causually un-connected events, this we will call space-like distance. At the Big Bang limit the first ("classical") approach should be transformed into the second ("quantum") one.

V. INERTIAL MASS IN QUANTUM MECHANICS DEFINED ON A DISCRETE SET

In standard quantum mechanics we start with the space of square-integrable functions $\mathrm{L}^2(\mathrm{R})$, the evolution operator $U(t)$, such that $\hat{U}(t)\hat{U}(t') = \hat{U}(t+t')$, and also some representation of canonical comutational relations. The latter, imply the existence of canonical momentum operator $\hat{p} = -\imath\hbar\partial/\partial x$ defined on a locally Euclidian (or Minkovskian) coordinate space. The coordinates are then used to construct a basis in the Hilbert space of state vectors, viz. $\psi(x) = \langle\psi|x\rangle$. The possibility of all these constructions is not self-evident for arbitrary discrete sets, but the basic principles are still applicable. Using the p-adic toy model, we are now going to show hereafter, how the concept of the mass can be derived in quantum mechanics of a discrete set.

Let $|i\rangle$ be a complete set of base observables, $|\psi(t)\rangle \in \mathcal{H}$ be a time-evolving state vector, then

$$C_i(t) = \langle i|\psi(t)\rangle \in \mathrm{L}^2(\mathrm{R})$$

is a state function. If a p-adic system is considered the basic vectors in $\mathcal{H}$ can be labeled by p-adic numbers $i \in Q_p$ and $\langle i|\psi(t)\rangle$ is the amplitude of the probability to find *an excitation of the i-th object.* The evolution operator can be written in usual form:

$$U_{ij}(t+\Delta t, t) = \delta_{ij} - \frac{1}{\hbar}\Delta t H_{ij}, \tag{13}$$

where $i, j \in Q_p$ and time is considered classically. Taking the scalar product of $\langle i|$ and the action of the operator (13) to the vector $|j\rangle$ one gets

$$\imath\hbar\frac{dC_i(t)}{dt} = \sum_j H_{ij}(t)C_j(t), \quad i, j \in Q_p,$$

or, using the convenient notation of p-adic integration

$$\imath\hbar\frac{dC(i)}{dt} = \int_{Q_p} H(i-j, t)C(j)dj. \tag{14}$$

Here we to make some physical asumptions about the Hamiltonian in (14). First, in the spirit of our geometrical interpretation, we assume that quantum transitions ocure between equidistant objects ($|x|_p = |y|_p$),i. e. they correspond to p-adic free particle motions (translations): $x' = x + a$, where $|x|_p = |a|_p$. Thus, in the simplest case we consider only the transitions within fixed p-adic circles. The system (14) can be then rewritten in the form

$$\imath\hbar\frac{dC_i}{dt} = \sum_{j=0}^{p-1} H_{ij}(t)C_j(t), \quad 0 \le i, j < p \tag{15}$$

($p > 2$ to get a physically sensible theory). Further, taking $p = 3$ to get the most simple model, we follow the consideration [19], and after some simple algebra arrive at the equaion

$$\imath\hbar \frac{dC(n)}{dt} = EC(n) - AC(n+1) - AC(n-1), \quad n \in \mathbb{Q}_3, \tag{16}$$

where the grownd state of the system can be chosen such, that $E = 2A$ [20].

Now, instead of a Taylor expansion, as was done in [19], we take a p-adic Fourier transform of the r. h. s. of (16). For convenience, we rewrite it in the form r. h. s. (16) $= -A(C(n+1) - C(n) + C(n-1) - C(n))$

$$\begin{aligned} C(n \pm 1) - C(n) &= \int_{Q_p} \big[\chi_p(k(n \pm 1)) - \chi_p(kn)\big]\tilde{C}(k)dk \\ &= \int_{Q_p} \chi_p(kn)\big[\chi_p(\pm k) - 1\big]\tilde{C}(k)dk \end{aligned}$$

So,

$$\begin{aligned} C(n+1) - 2C(n) + C(n-1) &= 2\int_{Q_p} \chi_p(kn)\Big[\frac{\chi_p(-k) + \chi_p(k)}{2} - 1\Big]\tilde{C}(k)dk \\ &= 2\int_{Q_p} \chi_p(kn)\big[\cos k - 1\big]\tilde{C}(k)dk \end{aligned} \tag{17}$$

As $n \in \mathbb{Z}_p$ is taken, the latter equation is non zero only for $\{k\}_p \neq 0$, i. e. $k \in \mathbb{Z}_p^{-1}$, hence k and n are dual to each other in the sense that $\mathbb{Z}_p \oplus \mathbb{Z}_p^{-1} = Q_p$. The equality $\frac{\chi_p(-k)+\chi_p(k)}{2} = \cos k$, always holds for the multiplication $k \cdot -1$ affects only the *integer* part of k and the factor $2\pi \times$ *integer* can be droped in the exponent. Expanding the cosine into the Taylor series

$$\cos k = \sum_{l=0}^{\infty} \frac{(-1)^l}{(2l)!} k^{2l}$$

and keeping the terms up to the first order in l we obtain a familiar form of the Schrödinger equation for a free particle

$$\imath\hbar \frac{dC(n)}{dt} = -A \int \chi_p(kn) k^2 \tilde{C}(k)dk. \tag{18}$$

It is important to emphasize here, that having started with a discrete system in (16) we arrive at the same Schrödinger equation as in continuous theory, but without any asumptions as to the existence of continuous space. If $n \in \mathbb{Z}_p$ labels the constituentsof the system, as was implied above, then $k \in \mathbb{Z}_p^{-1}$ can be understood as p-adic momentum. As in standard quantum mechanics, the energy of a free excitation here is $\frac{\hbar^2 k^2}{2m'}$, but it must be also equal to the energy term in the r. h. s. of the eq.(18):

$$\frac{\hbar^2 k^2}{2m'} = Ak^2.$$

Hence we can express the mass of the excitation in the quantum system (16) in terms of the probability amplitude A, and a dimensional constant $\hbar$:

$$m' = \frac{\hbar^2}{2A}. \tag{19}$$

The time t in eq.(18) is understood as an evolution parameter only, and we can choice $\hbar = 1$ system of units. *The mass of the excitation is therefore, up to a dimensional constant, just an iverse of transition probability amplitude.*

VI. CONCLUSION

The advantage of the above approach is that it is *coordinate-free* and thus no asumptions about the existence of any space before or beyond matter were used. We have shown that if there exists a set of objects which may be labeled by

p-adic numbers — the asumption is fairly general — we can introduce the mass as the inverse probability amplitude of transition between these objects, and further, the concept of metric also naturally arises.

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